A Model of Supply Chain with Possible Transshipmens Between Retails

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Mohamed Abu Gaben[1], Slobodan Krčevinac[2], Mirko Vujošević[2]
[1]*Al Quds Open University, Gaza, Palestine, mohamedgubeen@yahoo.com*
[2]*Faculty of Organizational Sciences, Beograd, mirkov@fon.bg.ac.yu*

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A supply chain with three stages - manufacturer, distributor and retail facilities - is considered. Stochastic demand is met on retail level. Retail facilities replenish their inventory periodically from the distribution centre and this is supplied from manufacturer on a periodical basis. Retail inventory is reviewed at the end of each period and new order is placed to the distribution centre. The following two supplying replenishment rules are considered: a) there is no transshipment between retails, and b) transshipment between retails is allowed. A mathematical model was developed for minimizing total inventory cost when demand is deterministic. The model was implemented using the AMPL modelling language. Optimal solutions for different scenarios, i.e. different sets of deterministic demand, are obtained using this model. These solutions are used as input data for new model which finds a compromise solution to minimizing the total regret. The total regret is defined as the sum of the differences between the total inventory cost obtained using compromise solution and the total inventory cost obtained using optimal solution for a given scenario. Experimental results show that implementing transshipment between retails may reduce total inventory cost.

1. Introduction

Inequalities in demand often result into the shortage of supplies in some retail facilities, while others report surpluses, regardless of their being part of the same supply chain. The classic rules of supply and the appropriate transportation models do not assume the redistribution of supplies among the retailers, that is, transshipment to the retail facility from another one. The shortage in supplies lasts as long as the delivery arrives from a supplier or from a distribution centre (DC). A logical issue that needs to be discussed is wheter a transshipment of supplies among retailers is possible and whether it pays. This paper is devoted to this very issue and offers a model to analyse various scenarios of the potential supplies redistribution.

The study of the phenomena in the supply chains [1,2] shows that the firm's performance is increasingly dependent on the performance of the chain it is part of. In order that the efficiency be improved, the issues of transportation, stocks and information support, as well as new rules for the chain operation are analysed. One modern approach is the implementation of transshipment and supply redistribution [3,4]. The modern information- communication infrastructure has allowed for the transshipment concepts to be implemented in dealing with the consumer commodity, and not, as it was earlier the case, only in dealing with relatively expensive products (e.g., automobiles) whose delivery, by a rule, takes time. The chain and the issue description discussed in this paper is presented in the next chapter. The third chapter brings a mathematical model, its translation into the algebraic modelling language AMPL [5], and the data used in the experiments. The findings and the analysis of these are presented in the fourth chapter, whereas the fifth chapter offers brief concluding remarks related to the research the findings of which are presented below.

2. Problem description and formulation

Redistribution is defined as a process of cross, lateral, transfer of stocks from one retail facility (RF) with a surplus of stocks to another RF that reports a shortage. The stocks redistribution among the RF is often a less expensive and a more convenient option compared to the increase in the number of deliveries from the supplier. It is one method to reduce the risk of the shortages in the product supply. Besides, redistribution increases the frequency of supplies and shortens the time of delivery which could otherwise be unacceptably long due to long distances between the suppliers and distribution centres, or due to the orders being extremely small. This is also the way to reduce the storage charges and the costs of stock-outs.

In case of the classical model, the RF orders are placed to the suppliers in advance, before it can be predisted what the demand will be like in the following period. Transshipment is the method of reallocation of stocks in accordance with the demand realized in the previous period. The transshipment of stocks helps reduce the costs of storage of defficient stocks in the sales facilities where the realized demand is not in accord with the planned one. Transshipment of stocks is considered to be an effective method of dealing with stohastic demand as well as of improving the performance and the reliability of the chain.

The stocks transshipment concept, however, requires that the communication and information exchange in the supply chain be increased. The Internet and other modern communication technologies ensure efficient ways of communication among the agents in the supply chain. Contrary to the traditional systems of business information exchange – EDI (Electronic Data Interchange) that only really big firms could afford, the information exchange mechanism provided by the Internet technologies and the mobile computer science is now available and affordable to small firms too. Therefore, the requirement that communication should be efficient for the purpose of the stocks transshipment concept implementation is today easy to satisfy via the Internet.

We will here analyse the impact the transshipment concept has upon the business policy of stocks ordering and replenishment. We will observe a single supplier system, with a centralized distribution centre and a large number of RFs, Figure 1. The supplier delivers periodically to the distribution centre, and the redistribution of stocks among the RFs is possible in the intervals between the two deliveries. The transshipment is reviewed at the end of each period and the redistributed products can be used to meet the demand in the periods to come. Contrary to the research so far, this model explicitely includes the time of delivery and observes the problem of a number of periods when the stocks transshipment can be used to meet the demand not only in the next, but also in any following periods.

The first step in the development of the model with a possible stocks transshipment is to define the way the supply chain functions. Here the following assumptions are made.

The assumptions:

1. One supplier meets the total demand of the RF network.
2. The supplier delivers to distribution centres in the network, which further deliver to the RF.
3. The location and number of RF are specified.
4. The capacities of distribution centres and of RF are infinite.
5. The transshipment of goods can be made between any two retail facilities.
6. If the RF reports the shortage of goods in the period *t*, the sales are considered as lost. Furthermore, there is an additional cost due to the loss of customer loyalty.
7. Each RF first uses its stocks to meet their own demend and only then, if requested, can send the surplus to other RS points.
8. Transshipments takes place at the end of each period, for the purpose of meeting the demand in the following one.
9. The supplier delivers to distribution centres on the basis of several (five) periods (i.e., at the beginning of every week, if a period is longer than a day), while the deliveries from distribution centres and the redistribution are shopped in all periods (i.e., daily).
10. The value of demand in the preceeding period serves to anticipate the demand in the period to come.
11. The surplus of stocks cannot be returned to the supplier.
12. The costs of keepng stocks can formally be viewed as costs of stocks self-redistribution.
13. There is no competition among the RS points.

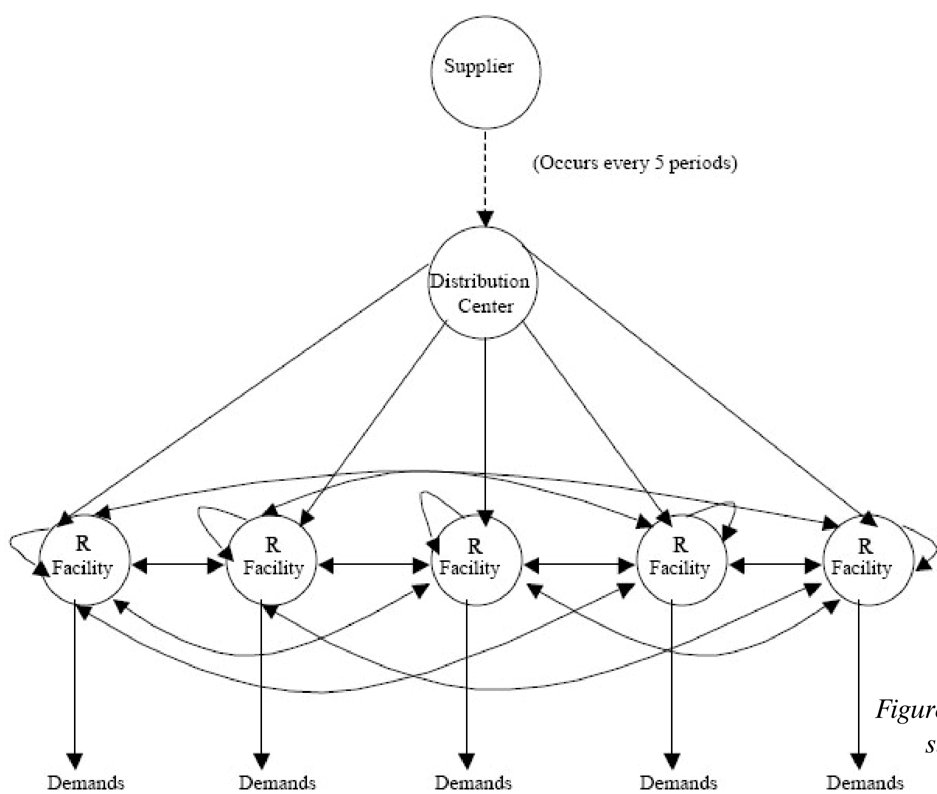


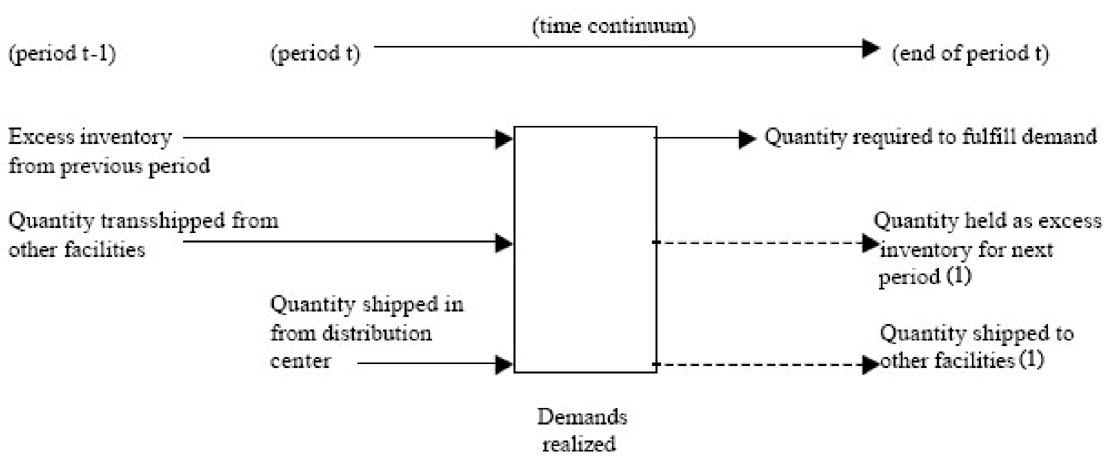
Figure 1. Topology of supply chain

In this network, the RFs receive goods before the demand has been met. Over time, the stocks in each RF are used to meet the demand in the respective period. At the end of the period, the surplus of supplies can be retained or redistributed to other sales facilities. In either situation the goods can be used to meet the demand in the subsequent period. Figure 2 presents the time flow with core events.

The original task set before the system designer and the planner is to define the policies of stock supply and redistribution in order to minimize the total costs incurred in the operations within the given supply chain. These costs include: fixed costs of supplies, delivery charges, costs of storage and costs of shortages. As regards the stohastic character of demand, the total costs are also random costs. This means that a solution that could best meet one demand, need not be optimal, may even prove to be a very unfavourable solution if another, different demand arises. Hence the approach to solving this problem is adopted that is based on experimenting and simulation of various scenarios. One scenario is defined by the number of periods and the respective values of demand. These values are previously defined randomly, according to the adopted function of the demand probability distribution. Then the optimum solution to stocks and transport management, that is, the optimum scenario solution is worked out for the generated values of demand, viewed as deterministic input data.

The more different scenarios, the more optimum solutions. In the end, the task is set to find a solution whose performances (costs) least deviate from the individual optimum costs of the concrete scenarios. This ultimate solution is a compromise and it need not be identical to any other individual optimum solutions obtained in analysisng a concrete scenario. The compromise solution, optimum in a way for all the scenarios viewed as a whole, is presented by the quantities to be periodically transported from the supplier to the distribution centres, from the distribution centres to each RF and by the value of the transshipment among the RFs.

Appendix B
Network Time Flow



(1) If a shortage exists in period t, neither excess inventory nor transshipments can occur.

Figure 2. Time flow in the supply chain

In order to examine the effectivity of the compromise solution, the function of the multiscenario model objective is defined as the total regret taken as a sum of regrets per scenario. The regret per scenario is in turn defined as the difference between the costs incurred when a compromise solution is applied to this scenario and the costs incurred in applying the optimum solution to the scenario.

3. Mathematical model

In formulating a multiscenario model we will use the notation adjusted to the requirements of the AMPL modelling language, [5].

3.1 Notation

Indices and sets

K – set of all the scenarios indexed by k,

W – set of all distribution centres indexed by w,

I – set of all RS points indexed by i and j,

T – set of time periods indexed by t ; $t = 1$ denotes the period 1.

Input parametres

n – number of time intervals,

s_w – unit cost of delivery from the supplier to the distribution centre w,

c_{wi} – unit cost of delivery from the distribution centre w to the RF i,
f_{ij} – unit cost of redistribution from RF i to RF j,
u – unit cost of shortage in supplies,
g_w – fixed costs added to the w distribution centre location,
d_{ik}^t – demand in RF i in the t period for the k scenario,
D – the factor ensuring that deliveries from the supplier are effected upon the expiry of five periods,
h_i – initial inventories in RF i,
q_t = {1, if the t period is the period in which the supplier delivers to distribution centres, 0, otherwise }
v_w – initial inventories in the distribution centre w

Managerial variables

X_{ijk}^t – the quantity redistributed from RF i to RF j in the period t for the scenario k;
Y_{wi}^t – the quantity delivered from the distribution centre w to RF i in the period t;
Z_w^t – the quantity delivered from the supplier to the distribution centre w in the period t;

Performances

O_k – the amount of minimum costs (optimum value) for the scenario k;
P_{ik}^t – the amount of shortage od stocks in RF i, at the end of the period t for the scenario k;
R_k – regret per scenario k;
U_{ik}^t = { 1 if there is a shortage in stocks in RF i in period t for the scenario k, 0 otherwise }

3.2. The TRANSSHIPMENT model

On the basis of the assumptions and the adopted notation, a TRANSSHIPMENT model was created to describe the supply chain from the supplier, to the distribution centre, to the retail facilities (RFs). The stohastic character of demand in the RFs is described using different scenarios and the regret concept described earlier in the paper. The TRANSSHIPMENT model is an optimisation model used to devise a compromise optimum solution to minimize the expected value of regret for a larger number of scenarios.

$$\textit{minimum} \quad \Sigma_k R_k \tag{1}$$

$$\textit{p.o.:} \quad R_k = (\Sigma_w \; g_w + \Sigma_i (h_i f_{ii} + \Sigma_t (uP_{ik}^t + \Sigma_w (s_w Z_w^t + c_{wi} Y_{wi}^t) + \Sigma_j f_{ij} X_{ijk}^t))) - o_k \qquad \forall\, k \tag{2}$$

$$\Sigma_w Y_{wi}^t + \Sigma_j X_{ijk}^{t-1} \geq d_{ik}^{t-1} \qquad \forall\, k, i, t \text{ in } 2..n \tag{3}$$

$$\Sigma_w Z_w^t \geq \Sigma_{t-5,t-1} \Sigma_i q^t d_{ik}^{t-1} \qquad \forall\, k, t \text{ in } 6..n \tag{4}$$

$$\Sigma_w Y_{wi}^1 + h_i - d_{ik}^1 - \Sigma_j X_{ijk}^1 + P_{ik}^1 = 0 \qquad \forall\, k, i \tag{5}$$

$$\Sigma_w Y_{wi}^t + \Sigma_j X_{jik}^{t-1} - d_{ik}^t - \Sigma_j X_{ijk}^t + P_{ik}^t = 0 \qquad \forall\, k, i, t \text{ in } 2..n \tag{6}$$

$$\Sigma_j X_{ijk}^t \leq D(1 - U_{ik}^t) \qquad \forall\, k, i, t \tag{7}$$

$$P_{ik}^t \leq d_{ik}^t U_{ik}^t \qquad \forall\, k, i, t \tag{8}$$

$$\Sigma_i Y_{wi}^t \leq v_w + \Sigma_{t2=1,t-1} (Z_w^{t2} - \Sigma_i Y_{wi}^{t2}) \qquad \forall\, w, t \tag{9}$$

$$Dq^t \geq Z_w^t \qquad \forall\, w, t \tag{10}$$

Model 1. TRANSSHIPMENT

In the above model the objective function is presented in (1) and (2) and expresses the total regret, where the formula (2) defines the the regret value for the k scenario. The constraint (3) reflects the assumption that demand is predicted on the basis of the previous period demand value. In the constraint (4) we imply that the amount of goods ordered from the supplier in each delivery period has to be based on the total demand incurred since the previous delivery period. The constraints (5) and (6) specify that the amount of goods delivered to one RF equals the demand in this facility plus the amount of stocks transshiped from this facility to others.

The constraint (7) refers to the fact that no transshipment is allowed from a given RF until its demand has been met. The constraint (8) says that the amount of shortage in one RF cannot be larger than the demand in that same facility. The constraint (9) does not allow the shipments from the distribution centre to be larger than the shipments received from the supplier. Finally, the constraint (10) says that the supplier can deliver goods to distribution centres only in set periods.

Prior to working this model out, it is necessary that optimum solution o_k for each individual scenario k be found. Hence a single-scenario model was created where the objective function is expressed by the total costs, while the constratints in the functioning of the supply chain are the same as in the previous model. The inputs to this model are the deterministic values of demand in each RF plus the shipment and storage charges. The model helped find the optimal solutions of o_k for each k scenario. These values were used as inputs for calculating the regret in the TRANSSHIPMENT model.

3.3 Data

In preparing the data we had to take several important characteristics of real issues into consideration. For example, it was important that the shortage costs be reasonably higher compared to shipment charges. If the costs of the shortage in supplies are too low, there will be no motivation to replenish the inventory by ordering from the supplier. Similarly, the data must reflect the realistic fact that in everyday practice it is cheaper for the RF to order goods directly from the distribution centre, rather than through another retail facility, which can be written as follows:

$$c_{wj} < c_{wi} + f_{ij}\,, \qquad \forall\ w, i, j$$

in addition, for the transshipments to be economically meaningful, it is necessary that the transshipment costs be lower than the storage charges in one RF and the shortages costs in another. This requirement can be written as follows:

$$f_{ij} < f_{ii} + u, \qquad \forall\ i\,, j$$

To simplify the presentation, we decided to model the demand in the RF as an independent random variable with a normal distribution where the means and variances for the facilities differ, however not much, since RF are supposed to operate in similar markets. Independence in this case means that the demand in one RF does not affect the demand in another. It is also assumed that the demand in one facility cannot be transferred to another.

To illustrate this supply chain we chose a single supplier, single distribution centre and five retail facilities (A, B, C, D, E) network. Table 1 shows the parametres used to generate demand in each of these objects.

RF	A	B	C	D	E
Mean	100	90	90	110	130
Standard deviation	10	10	8	10	15

Table 1. Demand in retail facilities

Using the random number method in the MS Excel, and on the basis of Table 1, we generated 50 random values of demand for each of the RFs. This fulfilled the needs of 5 different scenarios, 10 time periods per each of them. The scenarios helped model the unpredictability of demand and the potential variations in real demand.

3.4 AMPL model

The TRANSSHIPMENT model was translated into the computer executive variant using the AMPL modelling language. The database listing containing the AMPL model is the following:

AMPL Model1. TRANSSHIPMENT.mod

```
set SCENARIO; # scenarios
set DISTCENTR; # distribution centres
set MP; # retail facilities
set PERIOD;

param numperiods>=0;
param costsw {DISTCENTR} >=0;
param costwf {DISTCENTR, MP} >=0;
param costff {MP, MP} >=0;
param short>=0;
param fixed {DISTCENTR} >=0;
```

```
param demand {MP, PERIOD,SCENARIO} >=0;
param weight >=0;
param initial Inventory {i in MP}>=0;
param shipping period {t in PERIOD} binary;
param initial InventoriesDC{w in DISTCENTR}>=0;
param optimal {s in SCENARIO}>=0;

var shortamt {i in MP, t in PERIOD, s in SCENARIO}>=0;
var shipff {i in MP, j in MP, t in PERIOD, s in SCENARIO}>=0;
var shipwf {w in DISTCENTR, i in MP, t in PERIOD}>=0;
var shipsw {w in DISTCENTR, t in PERIOD}>=0;
var shortage {i in MP, t in PERIOD,s in SCENARIO} binary;
var regret {s in SCENARIO};

minimize total regret:  sum {s in SCENARIO} regret[s]; #objective function  (1)

subject to Regret per Each Scenario {z in SCENARIO}:
regret[z]= (sum {w in DISTCENTR} (fixed[w])+
sum {i in MP} (initial Inventories[i]*costff[i,i] +
sum {t in PERIOD} (short*shortamt[i,t,z]+
sum {w in DISTCENTR} (costsw[w]*shipsw[w,t]+costwf[w,i]*shipwf[w,i,t])+
sum {j in MP} costff[i,j]*shipff[i,j,t,z]))) - optimal[z]; # regret (2)

subject to Previous Demand1 {i in MP, t in 2..numperiods, s in SCENARIO}:
sum{w in DISTCENTR} shipwf[w,i,t] + sum{j in MP} shipff[j,i,t-1,s] >= demand [i,t-
1,s]; # constraint (3)

subject to Previous Demand2 {t in 6..numperiods, s in SCENARIO}:
sum {w in DISTCENTR} shipsw[w,t]>= sum{t-5..t-1, i in MP}
(Shipment period[t]*demand[i,t,s]); # constraing (4)

subject to Balance in Period1 {i in MP, s in SCENARIO}:
sum {w in DISTCENTR} shipwf[w,i,1] + initial Inventories[i] - demand[i,1,s] -
sum {j in MP} shipff[i,j,1,s] + shortamt[i,1,s]=0; # constraint (5)


subject to Balance {i in MP, t in 2..numperiods, s in SCENARIO}:
sum {w in DISTCENTR} shipwf[w,i,t]+sum {j in MP} shipff[j,i,t-1,s]
- demand[i,t,s] - sum {j in MP} shipff[i,j,t,s] + shortamt[i,t,s]=0; # constraint (6)

subject to shiplimit {i in MP, t in PERIOD, s in SCENARIO}:
sum {j in MP} shipff[i,j,t,s]<=weight*(1-shortage[i,t,s]); # constraint (7)

subject to shortagelimit {i in MP, t in PERIOD, s in SCENARIO}:
shortamt[i,t,s] <=demand[i,t,s] * shortage[i,t,s] ; # constraint (8)

subject to shiplimitfromDC {w in DISTCENTR, t in PERIOD}:
sum {i in MP} shipwf[w,i,t]<=initial InventoriesDC[w]+sum {t2 in 1..t-1}
(shipsw[w,t2] - sum{i in MP} shipwf[w,i,t2]); # constraint (9)

subject to ShipToDC {w in DISTCENTR, t in PERIOD}:
weight*Shipping period[t] >= shipsw[w,t]; # constraint (10)
```

The following listing is a database type .dat containing the input data for the AMPL Model1. TRANSSHIPMENT

AMPL Model1. TRANSSHIPMENT.dat

```
set SCENARIO:=1 2 3 4 5;
set WRHSE:= W;
set FACIL:= A B C D E;
set TIME:=1 2 3 4 5 6 7 8 9 10;

param numperiods:= 10;
param initialInventoryDC:= W 0;
param costsw := W 2;
param short:= 25;
param fixed:= W 10000;
param weight:= 10000;
param optimal := 1 74450
            2 78805
            3 81698
            4 79836
            5 82178;

param initInventories:=     A 0
                            B 0
                            C 0
                            D 0
                            E 0;

param costff (tr):   A B C D E:=
                            A 1 2 3 4 5
                            B 2 1 2 3 4
                            C 3 2 1 2 3
                            D 4 3 2 1 2
                            E 5 4 3 2 1;


param shipping period:= 1 1
2 0
3 0
4 0
5 0
6 1
7 0
8 0
9 0
10 0;
param costwf (tr): W:= A 1
B 1
C 1
D 1
E 1;
param demand :=
[*,*,1]:
1 2 3 4 5 6 7 8 9 10:=
A 85 107 91 96 89 131 109 114 89 123
B 75 97 81 86 79 60 82 84 84 82
C 71 91 79 97 95 67 99 71 103 96
D 97 118 102 111 125 84 116 116 118 104
E 110 142 118 131 153 94 119 131 108 127
[*,*,2]:
1 2 3 4 5 6 7 8 9 10:=
A 75 95 96 84 105 104 96 122 102 107
B 66 91 96 67 95 96 91 96 100 87
C 72 95 107 91 90 86 73 85 87 89
D 97 114 101 100 114 111 112 121 127 107
E 163 139 141 132 170 102 124 121 133 152
[*,*,3]:
1 2 3 4 5 6 7 8 9 10:=
A 93 113 103 77 106 114 98 116 107 106
B 90 96 103 92 81 80 101 75 91 94
C 99 87 88 94 101 81 99 89 77 76
D 110 123 123 100 125 90 123 115 115 106
E 145 123 139 141 156 148 154 121 123 158
[*,*,4]:
1 2 3 4 5 6 7 8 9 10:=
A 117 114 89 107 109 100 106 115 99 103
B 101 106 110 86 95 75 96 90 84 93
C 95 108 100 90 93 95 97 86 91 83
D 116 101 102 122 107 84 124 110 93 112
E 134 113 130 152 115 124 136 142 136 126
[*,*,5]:
1 2 3 4 5 6 7 8 9 10:=
A 98 97 96 110 103 114 114 96 100 131
B 89 104 72 102 84 95 87 72 88 103
C 85 98 96 89 87 89 95 86 90 81
D 101 103 102 101 105 120 110 115 104 100
E 104 165 95 146 133 153 118 163 130 109;
```

AMPL Model1. TRANSSHIPMENT.dat

4. Results of the experiment

The Model1.TRANSSHIPMENT.mod with Model1.TRANSSHIPMENT.dat input data was devised on the Intel Pentium 4 (Northwood), 2 GHz, 1GB DDR-SDRAM. A GLPSOL solver, component part of "Open Source" of the GLPK package (GNU Linear Programming Kit) was used, and this is meant for solving the problems of linear and mixed even number programming [6]. The package consists of a large number of GLPK API routines written in the C language and can be retrieved from the user application. The GLPSOL solver can be used autonomously for solving problems formulated in the following formats:

- LP/MIP model in GNU LP format,
- LP/MIP problem in fixed MPS format,
- LP/MIP problem in freeMPS format,
- LP/MIP problem in CPLEX LP format,
- LP/MIP model written in GNU MathProg modelling language.

The GNU MathProg modelling language is a subset of the AMPL, however, since it is open for general use, there are no limitations as to the number of variables nor limitations enforced by the AMPl – 300 x 300 student version.

We were concerned with solving three varieties of the model:

1. Model1.TRANSSHIPMENT that considers a possible distribution between the RFs.
2. Model2.NoTRANSSHIPMENT which considers the case identical to the previous one, however, without the possibility of transshipment, i.e., RF are independent from each other.
3. Model3.Deterministic which does not consider the randomness of demand as the two previous models do, using the scenario concept, but is otherwise identical to the previous model.

The results obtained are as follows:

1. The model had a total of 1815 variables, of which 250 even-number, and 250 binary, and 1026 constraints. There were 8390 non-zero values in the constraints matrix.
2. The optimum value for the objective function TotalRegret is 22896.
3. The total costs and the total regret for the three previous cases are shown in Table 2, where the %Difference is given as related to the costs of the deterministic case:

Table 2. Optimization results

	Total costs	Total Regret	% Difference
Transshipment allowed case	419863	22986	5.77 %
No transshipment case	425686	28719	5.23 %
Deterministic case	396967	-	-

The deterministic case was expected to display minimum costs, since the future demand is known precisely. In such circumstances the safety inventory is not necessary and this reduces the storage charges. Besides, planning helps eliminate the costs of shortages. The deterministic model is also the easiest problem to solve. Contrary to the deterministic model, the cases with and with no transshipment have to find compromise solutions.

In the case with an allowed transshipment among the RFs, the total costs are somewhat lower compared to the costs in the no transshipment case. A more thorough analysis of the two cases, however, can be conducted by observing the storage policy and the frequency of stock-outs. The results of the analysis are shown in Table 3.

Table 3. Optimization results analysis

	Number of shortages	Frequency of shortages	Maximum shortage amount	Average value of shortage amount
Transshipment allowed case	8	3,56 %	13	7,25
No transshipment case	15	6,67 %	40	12,8

A conclusion can be drawn that the number of shortages almost doubles when transshipment is not allowed, since the RF has to meet an increased demand in a given period and hence stores the inventory ordered for the future periods, and this may reasult in shortages in other RFs. On the other hand, in case the transshipment among the RFs is allowed, the RF in which the demand is lower than predicted can share the surplus with the facility reporting the shortage. In this case new transshipment costs emerge, however, the storage charges and the shortage costs are avoided. Also, the maximum and the mean values of the shortage amount is by far more favourable in the transshipment case than in the case without it.

Table 4 shows the differences between the average values of total inventory in one period in both cases under consideration.

Table 4. Optimization results analysis

	Average inventory quantity stored over a period
Transshipment allowed case	14,89
No transshipment case	27,83

As seen in Table 4, there is a significant difference in the inventory quantity stored in the RFs. This is an important argument in favour of introducing the transshipment type system if the RFs dispose of small warehouse capacities or if we deal with perishable or seasonal goods.

Another interesting characteristic for analysis is the behaviour of safety inventory, that is, to what extent the transshipment type systems reduce the amount of safety inventory due to risk pooling. Table 5 shows the results of this analysis.

Table 5. Delivery analysis

	Total delivery to DC	Total delivery to RF	Mean value of deliveries to RF	Standard deviation of quantity delivered
Transshipment allowed case	5549	4785	106,33	21,47
No transshipment case	5556	4895	108,78	23,21

On the basis of the data from Table 5 we can conclude that the quantities of goods delivered to DC or RFs are only slightly smaller in the transshipment allowed case. The explanation of this can be found in the model formulation. Namely, the model requires that the goods be ordered in accordance with the demand in the previous periods. If the model were based on the respective demand distributions, a more significant effects of risk pooling could be expected in the cases where transshipment is allowed. Similarly, if the input data for the demand distribution and the cost parametres were different, a significant effect of transshipment upon the safety inventory could be expected.

5. Conclusion

The model presented allows for the analysis of different supply policies, as well as different replenishment policies in the supply chain. The experimentally obtained findings with hypothetic but really possible data lead to the conclusion that the system with the transshipment in the supply chains may significantly reduce the costs in the stohastic demand cases, especially if the storage charges and shortage costs are high. It must, however, be pointed out that the systems with transshipment require a higher level of organization, trust and communication among the agents in the supply chains compared to the systems with no transshipment.

The model presented could, naturally, be improved in various directions – by the analysis of the model sensitivity to different cost parametres, as well as by the analysis of its implementation in various industries or in stocks management of different products. The model can also be extended by introducing a larger number of distribution centres and the problems of assigning the RFs to distribution centres. A crucially substantial improvement of the model would be an explicit introduction of the stohastic programming method instead of the scenario method, however, the introduction of a significantly larger number of scenarios would also prove to be interesting.

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